

Problem 1

If the initial temperature of the rod were

$$u(x, 0) = \sin \pi x \quad 0 \leq x \leq 1$$

and if the BCs were

$$u(0, t) = 0$$

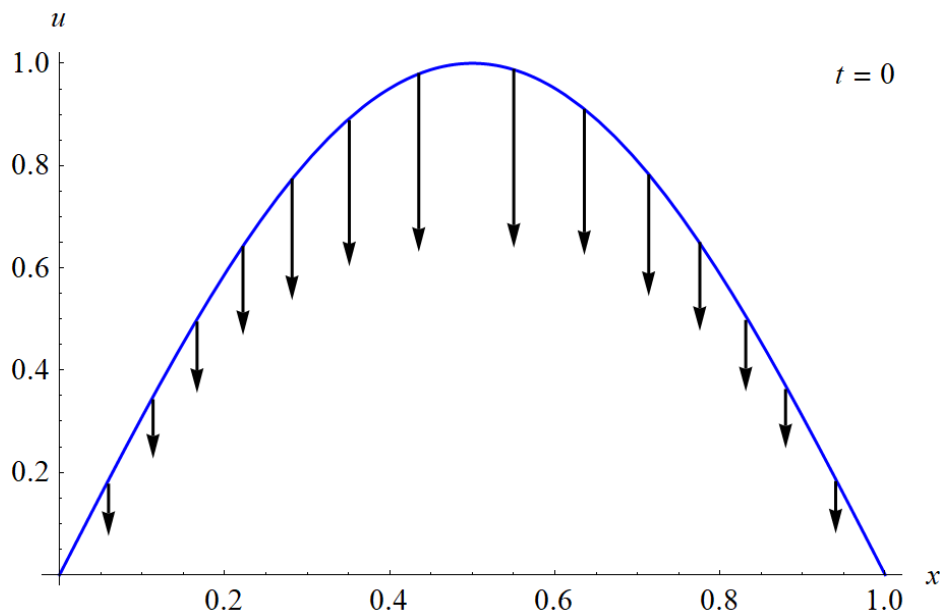
$$u(1, t) = 0$$

what would be the behavior of the rod temperature $u(x, t)$ for later values of time?

HINT Use the physical interpretation of the heat equation $u_t = \alpha^2 u_{xx}$.

Solution

The heat equation says that the rate of increase of u with respect to time is directly proportional to the curvature at any point on the line. For any initial temperature profile, then, the temperature will be decreasing at points of negative curvature and will be increasing at points of positive curvature. For the sine curve in this problem, then, the temperature will decrease at every point over time.



The general solution to the boundary value problem can be found with the method of separation of variables.

$$u(x, t) = e^{-\alpha^2 \pi^2 t} \sin \pi x.$$

This formula gives the temperature at every point on the rod ($0 < x < 1$) at all times ($t > 0$).

Snapshots of the temperature profile along the rod are shown for various times with $\alpha = 1$.

